Neutron and nuclear matter with 3N interactions

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Significance of nuclear and neutron matter results

• for the extremes of astrophysics: neutron stars, supernovae, neutrino interactions with nuclear matter

• microscopic constraints of nuclear energy-density functionals, next-generation Skyrme functionals

\[ \mathcal{E} = \frac{\tau}{2M} + A[\rho] + B[\rho] + C[\rho] |\nabla \rho|^2 + \cdots \]

• universal properties at low densities \( \rightarrow \) ultracold Fermi gases

**My focus:** Development of efficient methods to include 3N forces in microscopic many-body calculations of nucleonic matter
Reminder: Chiral EFT for nuclear forces

\[
\mathcal{O} \left( \frac{Q^0}{\Lambda^0} \right) \quad \mathcal{O} \left( \frac{Q^2}{\Lambda^2} \right) \quad \mathcal{O} \left( \frac{Q^4}{\Lambda^4} \right)
\]

<table>
<thead>
<tr>
<th>LO</th>
<th>NN</th>
<th>3N</th>
<th>4N</th>
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<tbody>
<tr>
<td>(c_1), (c_3), (c_4) terms</td>
<td>X</td>
<td>-</td>
<td>-</td>
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<tr>
<td>(c_D) term</td>
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<tr>
<td>(c_E) term</td>
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large uncertainties in coupling constants at present:

\[
c_1 = -0.9^{+0.2}_{-0.5}, \quad c_3 = -4.7^{+1.5}_{-1.0}, \quad c_4 = 3.5^{+0.5}_{-0.2}
\]

Meissner et al., Machleidt
Chiral 3N interaction as density-dependent two-body interaction

antisymmetrized 3N interaction (at N^2LO) in neutron matter:

\[ V^{3N} = \frac{1}{2} \left( \frac{g_A}{2f_\pi} \right)^2 \sum_{i \neq j \neq k} A_{ijk} \frac{(\sigma_i \cdot q_i)(\sigma_j \cdot q_j)}{(q_i^2 + m_\pi^2)(q_j^2 + m_\pi^2)} \left[ -\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2} q_i \cdot q_j \right] \]

\[ V^{3N} = \text{antisymmetrized 3N interaction (at N^2LO) in neutron matter:} \]

\[ C_4, C_D, \text{ and } C_E \text{ terms vanish in neutron matter} \]

\[ \text{in nuclear matter all terms contribute} \]
Chiral 3N interaction as density-dependent two-body interaction

antisymmetrized 3N interaction (at $N^2$LO) in neutron matter:

$$V^{3N} = \frac{1}{2} \left( \frac{g_A}{2f_\pi} \right)^2 \sum_{i \neq j \neq k} A_{ijk} \frac{(\sigma_i \cdot q_i)(\sigma_j \cdot q_j)}{(q_i^2 + m^2_\pi)(q_j^2 + m^2_\pi)} \left[ -\frac{4c_1 m^2_\pi}{f^2_\pi} + \frac{2c_3}{f^2_\pi} q_i \cdot q_j \right]$$

$C_4$, $C_D$, and $C_E$ terms vanish in neutron matter

in nuclear matter all terms contribute

Basic idea: Sum one particle over occupied states in the Fermi sea

$$\overline{V}^{3N} = \sum_{q,\sigma} V^{3N} n(k_F - q)$$
Chiral 3N interaction as density-dependent two-body interaction

antisymmetrized 3N interaction (at N^2LO) in neutron matter:

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\[ V^{3N} = \begin{array}{c}
\pi \quad \pi \\
\pi \\ \pi 
\end{array} - \begin{array}{c}
\pi \\
\pi \\
\pi 
\end{array} - \begin{array}{c}
\pi \\
\pi \\
\pi 
\end{array} + \begin{array}{c}
\pi \\
\pi \\
\pi 
\end{array} + \begin{array}{c}
\pi \\
\pi \\
\pi 
\end{array}
\]

\[ C_4, C_D \] and \[ C_E \] terms vanish in neutron matter
in nuclear matter all terms contribute

Provides 3N corrections to free space NN interaction:
Operator form of $V^{3N}$ in neutron matter

general momentum dependence: $\overline{V}^{3N} = \overline{V}^{3N}(\mathbf{k}, \mathbf{k}', \mathbf{P})$

P-dependence only weak, evaluate for $\mathbf{P} = 0$:

$$\overline{V}^{3N}_{P=0} = \frac{1}{2} \left( \frac{g_A}{2f_\pi} \right)^2 \left[ -\frac{4c_1m^2}{f^2_\pi} A(\mathbf{k}, \mathbf{k}') + \frac{2c_3}{f^2_\pi} B(\mathbf{k}, \mathbf{k}') \right]$$

$$B(\mathbf{k}, \mathbf{k}') =$$

$$-\frac{1}{3} \left\{ \frac{\rho(k, k')(k + k')^4}{((k + k')^2 + m^2_\pi)^2} + \frac{2}{3} B_1^s(k, k') - B_1^s(k, -k') - (B_2^s(k, k') + B_2^s(k', k)) \right\}$$

$$+ \frac{1}{3} (\sigma \cdot \sigma') \left\{ \frac{2}{3} \frac{\rho(k, k')(k - k')^4}{((k - k')^2 + m^2_\pi)^2} + \frac{1}{3} \frac{\rho(k, k')(k + k')^4}{((k + k')^2 + m^2_\pi)^2} \right.$

$$+ B_1^s(k, -k') - \frac{1}{3} \left[ B_2^s(k, k') + B_2^s(k', k) \right] - \frac{2}{3} \left[ B_2^s(k, -k') + B_2^s(k', -k) \right] \left. \right\}$$

$$+ \frac{2}{3} \left[ \frac{\rho(k, k')(k + k')^2 S_{12}(k + k')}{((k + k')^2 + m^2_\pi)^2} - \frac{\rho(k, k')(k - k')^2 S_{12}(k - k')}{((k - k')^2 + m^2_\pi)^2} \right]$$

$$+ \frac{2}{3} \sigma^a \sigma^b \left[ B^t_{ab}(k, k') - B^t_{ab}(k, -k') + B^t_{ab}(k', k) - B^t_{ab}(k', -k) \right]$$

$$+ \frac{1}{3} i (\sigma^a + \sigma'^a) \left[ B^v_{a}(k, k') - B^v_{a}(k, -k') \right]$$
Operator form of $V^{3N}$ in neutron matter

General momentum dependence: $V^{3N} = V^{3N}(k, k', P)$

P-dependence only weak, evaluate for $P = 0$:

$$V^{3N}_{P=0} = \frac{1}{2} \left( \frac{g_A}{2f_\pi} \right)^2 \left[ -\frac{4c_1m^2}{f^2_\pi} A(k, k') + \frac{2c_3}{f^2_\pi} B(k, k') \right]$$

$B^s_1(k, k')$

$$B^s_1(k_1, k_2) = \int \frac{d^3q}{(2\pi)^3} n(q) f_R(\Lambda_{3N}, q, k_1) f_R(\Lambda_{3N}, q, k_2) \frac{((k_1 + q) \cdot (k_2 + q))^2}{((k_1 + q)^2 + m^2_\pi)((k_2 + q)^2 + m^2_\pi)}$$

- neglect P-dependence in the following, set $P=0$
- in fixed-P approximation $V^{3N}$ matrix elements have the same form like genuine free-space NN matrix elements
- straightforward to incorporate in existing many-body schemes
Partial wave matrix elements \((\Lambda_{3N} = 2.0 \text{ fm}^{-1})\)

- non-trivial density dependence
- \(\overline{V}_{3N}(k, k'; 1\, S_0) \sim k_F^4 \sim \rho^{4/3}\)
- dominant central contributions
- non-central tensor and spin-orbit components

KH and A. Schwenk arXiv:0911.0483
Equation of state (EOS):
Many-body perturbation theory

central quantity of interest: energy per particle \( E/N \)

\[ H(\Lambda) = T + V_{\text{NN}}(\Lambda) + V_{\text{3N}}(\Lambda) + \ldots . \]

- for low momentum interactions no resummation of diagrams necessary
- self-consistent single-particle propagators \( \rightarrow \) thermodynamic consistency
Neutron matter:
EOS (first order), Test of fixed-P approximation

\[ E_{\text{full}}^{(1)} = V_{\text{NN}} + V_{3N} \]

\[ E_{\text{eff}}^{(1)} = V_{\text{NN}} \]

relative difference of
3N contributions only \( \sim 3\% \)

P-independent effective NN interaction is a very good approximation

KH and A.Schwenk arXiv:0911.0483
Neutron matter: EOS (second order)

- reduced cutoff dependence at 2nd order
- self-energy effects small
- system is perturbative for low-momentum interactions
Neutron matter: EOS (second order)

- energy sensitive to $C_3$ variations
- uncertainty due to coupling constants much larger than cutoff variation

$$c_1 = -0.9^{+0.2}_{-0.5}, \quad c_3 = -4.7^{+1.5}_{-1.0}, \quad c_4 = 3.5^{+0.5}_{-0.2}$$

KH and A. Schwenk arXiv:0911.0483
Symmetric nuclear matter: First order results

\[ E_{\text{full}}^{(1)} = E_{\text{NN}}^{(1)} + V^{NN} + E_{\text{3N,full}}^{(1)} \]

\[ E_{\text{eff}}^{(1)} = E_{\text{NN}}^{(1)} + V \]

- P-independent effective interaction also efficient for symmetric nuclear matter
- large 3N contributions, lead to saturation at HF level
- tensor force much stronger in SNM
Symmetric nuclear matter

- 3N forces crucial for saturation
- cutoff dependence at 2nd order significantly reduced
- couplings $c_D$ and $c_E$ fitted to $E_{3\text{H}} = -8.482 \text{ MeV}$ and $r_{4\text{He}} = 1.95 - 1.96 \text{ fm}$
- 3rd order pp and hh contributions small

Bogner, Furnstahl, Schwenk, Nogga, KH; in preparation
Symmetric nuclear matter

Improvements:
• full treatment of double exchange terms
• self-consistent single-particle self-energies
• correction of combinatorial factors

Bogner, Furnstahl, Schwenk, Nogga; arXiv:0903.3366

\[ E_{\text{NN+3N,eff}}^{(2)} \]

\[ r_{4\text{He}} = 1.95 - 1.96 \text{ fm} \]
Nuclear matter: Uncertainties due to coupling constants and RG scheme

Entem/Machleidt (EM):

\[ c_1 = -0.81 \text{ GeV}^{-1} \]
\[ c_3 = -3.20 \text{ GeV}^{-1} \]
\[ c_4 = +5.40 \text{ GeV}^{-1} \]

Rentmeester et al. (RM):

\[ c_1 = -0.76 \text{ GeV}^{-1} \]
\[ c_3 = -4.78 \text{ GeV}^{-1} \]
\[ c_4 = +3.96 \text{ GeV}^{-1} \]

- uncertainty of about 3.5 MeV in E/A at saturation density
- reasonable saturation properties
- improved constraints of \( C_i \) couplings necessary!
Pairing gap in semi-magic nuclei

Three-body mass difference:

\[ \Delta^{(3)}(N) = \frac{(-1)^N}{2} [E(N + 1) - 2E(N) + E(N - 1)] \]

repulsive 3N contributions lead to suppression of the pairing gap

Lesinski, Duguet, Schwenk, KH in preparation